Assignment 8

Coverage: 16.2, 16.3 in Text.

Exercises: 16.2 no 10, 12, 15, 21, 22, 25, 27, 29, 30, 32, 36, 43, 46. 16.3 no 29, 31, 32.

Hand in 16.2 no 36, 43; 16.3 no 31 by March 23.

Supplementary Problems

1. A region is called star-shaped if there is a point O inside so that the line segment connecting any point in this region to O lies completely in this region. Show that the compatibility condition (3.8) is also sufficient for the existence of a potential for the vector field $\mathbf{F}$ in a star-shaped region. Hint: Modify the proof of Theorem 3.4 slightly.

Exercises 16.2

Work, Circulation, and Flux in the Plane

36. Flux across a triangle Find the flux of the field **F** in Exercise 35 outward across the triangle with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$.

Vector Fields in the Plane

43. Unit vectors pointing toward the origin Find a field $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ in the xy-plane with the property that at each point $(x, y) \neq (0, 0)$, **F** is a unit vector pointing toward the origin. (The field is undefined at $(0, 0)$.)

Exercises 16.3

Applications and Examples

31. Evaluating a work integral two ways Let $\mathbf{F} = \nabla(x^3y^2)$ and let C be the path in the xy-plane from $(-1, 1)$ to $(1, 1)$ that consists of the line segment from $(-1, 1)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(1, 1)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ in two ways.

a. Find parametrizations for the segments that make up C and evaluate the integral.

b. Use $f(x, y) = x^3y^2$ as a potential function for **F**.